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NEUTRINO PHYSICS AND THE MIRROR WORLD*

R. R. VOLKAS
School of Physics, Research Centre for High Energy Physics, The University of Melbourne
Parkville 3052, Australia
E-mail: r.volkas@physics.unimelb.edu.au

ABSTRACT

Improper Lorentz Transformations can be retained as exact symmetries of Nature if the particle content and gauge group of the Standard Model are doubled. The resulting "Exact Parity Model (EPM)" sees each ordinary particle paired with a mirror analogue. If neutrinos have mass and if they mix, then the EPM predicts that each ordinary neutrino will be maximally mixed with its mirror neutrino partner. This provides a very simple explanation for the very large mixing angle observed for atmospheric muon neutrinos by SuperKamiokande and other experiments. Maximal mixing for electron neutrinos is also well motivated by the solar neutrino problem. If small interfamily mixing is switched on, then the LSND anomaly can also be accomodated by the EPM. The EPM thus provides a unified, simple and to some extent predictive framework for explaining all of the anomalous neutrino data. This talk will briefly review the EPM or mirror neutrino scenario.

1. Full Lorentz Invariance: Proper and Improper

For many decades there was a strong theoretical and/or aesthetic prejudice for fundamental physical laws that were symmetric under both spatial and temporal reflection (parity and time-reversal invariance). When the $V - A$ character of weak interactions was established in the late 1950's, exact parity invariance was apparently empirically falsified.[1,2] Soon after, the discovery of CP violation apparently falsified exact time-reversal invariance.[3] It seemed as if only the Proper Lorentz Transformations were true symmetries of Nature.

However, Lee and Yang recognised from the outset that parity can be retained as an exact symmetry, despite the $V - A$ character of weak interactions, provided that the ordinary particle spectrum is doubled.[2] From a modern theoretical and phenomenological perspective this requires every ordinary lepton, quark, gauge boson and Higgs boson to be paired with a mirror analogue.[4] Since the ordinary and mirror particle sectors are but weakly coupled to each other, the resulting scenario is phenomenologically viable. The violation of parity invariance is therefore, remarkably, still an open question. A simple argument to be presented below shows that if the above type of exact parity symmetry exists in Nature, then so necessarily does a form of time reversal invariance. So, it is possible for the full Lorentz Group to be a

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completely unbroken symmetry of Nature![4]

Of particular interest is the fact that the observed atmospheric and solar neutrino anomalies may be the first experimental manifestation of the mirror matter sector (mirror neutrinos to be specific).[5,6,7] In order to further test this proposal, neutral current based measurements probing both the atmospheric and solar anomalies are vital. Such measurements will determine whether the relevant ordinary neutrinos are transforming into other ordinary neutrinos, or into something more exotic such as mirror or sterile neutrinos.

The gauge theoretic construction of a theory with exact parity symmetry is very easy to understand.[4,7] Consider a theory defined by a parity *violating* Lagrangian $\mathcal{L}$ which has gauge group G. This theory may, for instance, be the minimal Standard Model, or, more pertinently, the Standard Model augmented by nonzero neutrino masses and mixings. For every field ψ in $\mathcal{L}$ introduce a mirror or parity partner ψ'. For spin-1/2 fields this requires of course that the ψ' have opposite chirality to the ψ. The fields ψ' are singlets under G but transform under a gauge group G' which is isomorphic to G, while the ψ's are correspondingly required to be singlets under G'. The fields ψ and ψ' are placed into identical multiplets under their respective gauge groups G and G', and the discrete parity symmetry (schematically $\psi \leftrightarrow \psi'$) is enforced. The resulting Lagrangian is

$$\mathcal{L}_{\rm total}(\psi, \psi') = \mathcal{L}(\psi) + \mathcal{L}'(\psi') + \mathcal{L}_{\rm int}(\psi, \psi'), \tag{1}$$

where $\mathcal{L}'$ is exactly the same function of the ψ' fields as $\mathcal{L}$ is of the ψ fields.[a] The extremely important interaction term $\mathcal{L}_{\rm int}$ describes any gauge and parity invariant renormalisable coupling terms between the ordinary and mirror sectors. The above procedure was first carried out for the minimal Standard Model in the first paper quoted under Ref.[4], where it was shown that parity was a symmetry of the vacuum as well as the Lagrangian for a large region in Higgs potential parameter space. We will focus on this parameter space region from now on.[b] We call the resulting theory the "Exact Parity Model (EPM)".

The ordinary and mirror sectors are coupled by gravitation and $\mathcal{L}_{\rm int}$. The gravitational coupling is very interesting from the point of view of cosmology and the dark matter problem,[9] but will not be further discussed here. The nongravitational effects in $\mathcal{L}_{\rm int}$ in general feature photon – mirror photon, Z – mirror Z and Higgs – mirror Higgs mixing. These particles are singled out because they are neutral under the electromagnetic and colour forces and their mirror analogues, so there are no exact conservation laws to prevent mixing. (Electron – mirror electron mixing is forbidden by both ordinary and mirror electric charge conservation, for instance.) Unfortunately, cosmological constraints from Big Bang Nucleosynthesis make it unlikely that

[a]The dependence of the Lagrangian on first derivatives of the fields is of course understood.

[b]By extending the Higgs sector, it is possible to spontaneously break the parity symmetry together with the electroweak and mirror-electroweak gauge symmetries.[8]

these effects will be seen in the laboratory.[10]

We now come to the crux of the matter: *since neutrinos and mirror neutrinos are electrically neutral and colourless, they will in general mix if they also have nonzero masses.* Furthermore, we will see in the next section that the exact parity symmetry forces the mixing angle between an ordinary neutrino and its mirror partner to be the maximal value of $\pi/4$.

I close this section with two brief comments. (i) Let the exact parity symmetry be denoted by P'. Note that it is different from the usual (broken) parity symmetry P.[c] However, standard CPT is still of course an exact symmetry of the theory. We can therefore define a non-standard time-reversal invariance T' through $CPT = P'T'$ that must necessarily be exact if P' is exact. The full Lorentz Group, including all Improper Transformations, is thus a symmetry of the theory.[4] (ii) It is amusing to compare the 'exact parity' idea with spacetime supersymmetry. Both extend the Proper Lorentz or Poincaré Group, and both require degree-of-freedom doubling. A crucial difference, though, is that phenomenology forces spacetime supersymmetry to be broken. The resulting proliferation of soft-supersymmetry breaking parameters has no analogue in the EPM.

2. Phenomenology of Mirror Neutrinos

Under the exact parity symmetry P', an ordinary neutrino field ν_α (where $\alpha = e, \mu, \tau$) transforms into its mirror partner field ν'_α as per

$$\nu_{\alpha L} \to \gamma_0 \nu'_{\alpha R}. \tag{2}$$

From basic quantum mechanics, we know that the exact P' symmetry forces the parity eigenstates to also be mass eigenstates. In the absence of interfamily mixing, this means that the two mass eigenstates $\nu_{\alpha\pm}$ per family take the form

$$|\nu_{\alpha\pm}\rangle = \frac{1}{\sqrt{2}} \left(|\nu_\alpha\rangle \pm |\nu'_\alpha\rangle \right). \tag{3}$$

The positive and negative parity states, $\nu_{\alpha+}$ and $\nu_{\alpha-}$ respectively, in general have arbitrary masses. The oscillation parameter $\Delta m^2_{\alpha+\alpha-} \equiv |m^2_{\nu_{\alpha+}} - m^2_{\nu_{\alpha-}}|$ is therefore free. The mixing angle, however, is forced by P' symmetry to have the maximal value of $\pi/4$.[5,6,7]

Interestingly, SuperKamiokande and other experiments have observed a disappearence of atmospheric muon-neutrinos in a manner which favours maximal mixing with another flavour ν_x.[11] Current results preclude $x = e$, but allow both $x = \tau$ and $x = s$[12] (where s stands for "sterile"). It is natural in the EPM to identify ν_x with the effectively sterile mirror muon-neutrino ν'_μ.[6,7] The maximal mixing angle for

[c]For instance, the left-handed electron is transformed into the right-handed mirror-electron by P', whereas it is transformed into the right-handed electron by P.

the $\nu_\mu - \nu'_\mu$ subsystem is a simple and characteristic prediction of the EPM that is strongly supported by experiment.[d] The $\Delta m^2_{\mu+\mu-}$ oscillation parameter is adjusted to agree with the measurements. This requires it to be in the $10^{-3} - 10^{-2}$ eV^2 range.[11,14]

The experimental discrimination between $\nu_x = \nu_s$ and $\nu_x = \nu_\tau$ is a vital further test of this proposal. Hope for progress in this area in the immediate future lies with SuperKamiokande atmospheric neutrino data and the K2K long baseline experiment.[15] The basic requirement is a neutral current measurement, since ν_τ is sensitive to this interaction while ν_s and ν'_μ are not. SuperKamiokande has quoted a measured value for the atmospheric neutrino induced π^0/e ratio (see Ref.[11] for the present status) that cannot discriminate between the two possibilities because of a large theoretical uncertainty in the π^0 production cross-section. The measurement of this cross-section by the K2K long baseline experiment is thus of great importance. This may allow a discrimination based on a zenith-angle averaged atmospheric neutrino induced π^0/e ratio by SuperKamiokande within about a year from the time of writing.[16] It should be noted, however, that the $\nu_\mu \to \nu'_\mu\,(\nu_s)$ case predicts that the actual π^0/e ratio will be about 0.8 times the no-oscillation or $\nu_\mu \to \nu_\tau$ expectation.[17] If the SuperKamiokande central value were to be around 0.9, then the remaining systematic error would still be too large to discriminate between the possibilities. A cleaner discrimination, which however requires significantly greater statistics, lies in the future through the π^0 up-down asymmetry.[18] The K2K experiment could in principle discriminate between the two possibilities on its own by comparing the neutral- to charged-current rates at the near and far detectors. However, inadequate statistics at the far detector (SuperKamiokande) may preclude a useful result. However, provided $\Delta m^2_{\mu+\mu-}$ is sufficiently large, it should at the very least confirm ν_μ disappearence. Looking slightly further into the future, the long baseline experiment MINOS and the proposed CERN–Gran-Sasso long baseline experiments should provide important information.[19]

The solar neutrino anomaly provides further motivation for the maximal mixing feature of the EPM.[5,7] Consider the maximally mixed $\nu_e - \nu'_e$ subsytem in the zero interfamily mixing limit. For the $10^{-3} \lesssim \Delta m^2_{e+e-}/\text{eV}^2 \lesssim 10^{-10}$ range, the maximal $\nu_e \to \nu'_e$ oscillations are consistent with disappearence experiments and lead to an energy-independent day-time solar neutrino flux reduction by 50%. This is consistent with four out of the five solar event rate meansurements relative to the latest standard solar model calculations.[20] (The Chlorine experiment sees a greater than 50% deficit.) Since this talk was given, Guth et al.[21] have emphasised that the night-time oscillation-affected solar neutrino rate differs from the day-time rate, even if the vacuum mixing angle is maximal. This leads to some energy-dependence in the night-time flux suppression, and provides an interesting further test. Preliminary calculations show that the day-night asymmetry for the $\nu_e \to \nu'_e$ case (or

[d] For attempts to explain the large mixing angle in the case of ν_x identified as ν_τ see, for instance, Ref.[13].

the $\nu_e \rightarrow \nu_s$ case with maximal mixing) is potentially observable for the range $6\times10^{-8} \lesssim \Delta m^2_{e+e-}/\text{eV}^2 \lesssim 2\times10^{-5}$, with the range $2\times10^{-7} \lesssim \Delta m^2_{e+e-}/\text{eV}^2 \lesssim 8\times10^{-6}$ already disfavoured by the data.[22]

The future KAMLAND experiment will probe the $10^{-3} \lesssim \Delta m^2_{e+e-}/\text{eV}^2 \lesssim$ few $\times$ 10^{-5} regime by looking for $\overline{\nu}_e$ disappearence.[23e] Another extremely important future test is the neutral to charged current event rate ratio that will be measured by SNO.[25] The mirror electron-neutrino ν'_e is effectively a sterile flavour, so SNO should measure the "standard" value for this ratio.

If $\Delta m^2_{e+e-}/\text{eV}^2$ is in the $10^{-10} - 10^{-11}$ range, then "just-so" oscillations result.[26] One amusing possibility[22] is the following: as Δm^2_{e+e-} is reduced from the range considered in the previous paragraph into the just-so regime, the energy at which the averaged oscillations give way to coherent just-so behaviour decreases. For some value, this transition will happen within the energy range probed by SuperKamiokande. This could possibly be the origin of the mysterious high-energy spectral feature reported by SuperKamiokande![20] (This type of idea was first examined in the context of ν_e oscillations into an active flavour in Ref.[27].)

So, putting the above in a nutshell, we have KAMLAND probing the high range for Δm^2_{e+e-}, the day-night asymmetry being used in the intermediate range, and just-so signatures such as seasonal variation and Boron neutrino energy spectrum distortion probing the low Δm^2_{e+e-} regime. The range between about 6×10^{-8} eV^2 and the beginning of the just-so region appears to have no characteristic signature other than the 50% energy independent flux suppression. Furthermore, the crucial neutral current measurement at SNO will test the general idea that solar neutrinos are disappearing into sterile states of some sort for the whole Δm^2_{e+e-} range of interest.

The above analysis saw interfamily mixing set to zero. Certainly, small interfamily neutrino mixing is well motivated by the small mixing observed for the quark sector. However, it is unlikely that this mixing exactly vanishes. I will now comment on three possible consequences of interfamily mixing.

First, the LSND anomaly[28] can be trivially accomodated within the EPM by switching on $\nu_e - \nu_\mu$ mixing with the appropriate parameter choices.[7] The LSND parameter regime does not significantly modify the solar and atmospheric neutrino scenario discussed above.

Second, the solar neutrino flux depletion can be due to an amalgam of vacuum $\nu_e \rightarrow \nu'_e$ oscillations and MSW interfamily transitions.[29] This leads to characteristic energy-dependent flux depletions depending on the precise oscillation parameter range chosen. Further, the neutral to charged current induced event rate ratio to be measured by SNO can take on values intermediate between the extreme cases of $\nu_e \rightarrow \nu_{\text{active}}$ only and $\nu_e \rightarrow \nu_s$ only.

Third, it turns out that small interfamily mixing is well motivated from cosmology,

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[e] The $\nu_e \rightarrow \nu'_e$ mode also has potentially observable consequence for atmospheric ν_e's for this parameter range.[24]

a topic I very briefly review in the next section.

3. Cosmology

The tale of how neutrino oscillations affect early universe cosmology is long and complicated. I will pass over it lightly here, just for the sake of completeness, without much in the way of explanations. Please consult, for example, Refs.[30,31,32,33] for further details.

Cosmology and ordinary-mirror (and ordinary-sterile) neutrino oscillations present challenges to each other. On the one hand, it had long been thought that sterile neutrinos ought to mix but weakly with ordinary neutrinos lest the reasonably successful Big Bang Nucleosynthesis (BBN) predictions be spoiled. In particular, it was thought that a $\nu_\mu \to \nu_s$ solution to the atmospheric neutrino problem would have necessarily implied the thermal equilibration of the sterile flavour prior to the BBN epoch, and thus would have increased the expansion rate of the universe. Recall that the expansion rate of the universe during BBN is driven by the relativistic degrees of freedom in the plasma, with "neutrino flavour number N_ν" being a convenient measure. In the minimal Standard Model $N_\nu = 3$, while one thermally equilibrated sterile flavour in addition to the ordinary neutrinos produces $N_\nu = 4$. There is some confusion in interpreting primordial element abundance data at present, but it is arguable that $N_\nu < 4$ is preferred.[34] So, it had been thought that a large region of active-sterile oscillation parameter space was at least disfavoured by BBN. This problem was seen to be much more acute for the EPM than for models with a single extra sterile state, because of the *three* mirror neutrino flavours as well as the mirror photons, electrons and positrons. Prior wisdom would have concluded that the EPM ruined BBN and was therefore unlikely to be true. Thus cosmology challenged sterile and mirror neutrino models.

On the other hand, the discovery of relic neutrino asymmetry amplification, driven by the ordinary-mirror or ordinary-sterile neutrino transitions themselves, showed that the previous pessimism was misplaced: a very natural mechanism for reconciling BBN with sterile or mirror neutrinos, born out of the apparently problematic neutrino scenario itself, actually existed all along but had been missed.[31] The basic point is that large relic neutrino asymmetries (neutrino chemical potentials) will, in a certain large region of oscillation parameter space, be inevitably created via a positive feedback process from the tiny CP asymmetry (baryon asymmetry for instance) of the high-temperature background plasma. The large matter (Wolfenstein) effective potentials so induced then damp further ordinary-mirror or ordinary-sterile transitions and lead to quite acceptable BBN predictions (for the appropriate region of parameter space).

The full story of BBN in the presence of ordinary-mirror/sterile transitions is complicated because many different oscillation modes are in general involved. We have discussed above how the excitation of mirror or sterile neutrinos prior to BBN

increases the expansion rate as quantified through N_ν. But there is another important effect: a fairly large electron neutrino asymmetry will be created before and during BBN given appropriate oscillation parameters. This asymmetry will directly affect BBN reaction rates and will alter the primordial Helium abundance so as to mimic *either* a negative *or* a positive contribution to the effective neutrino number during BBN. A detailed numerical calculation is often necessary to determine the final BBN outcome. Such calculations have been performed for a couple of models featuring a single sterile flavour.[32] They have demonstrated that strong ordinary-sterile neutrino mixing can be reconciled with BBN for realistic sterile neutrino models via the interesting physics just discussed.

The first full analysis of neutrino asymmetry evolution and BBN in the Exact Parity Model was completed after this Symposium.[33] It demonstrated that the EPM scenario outlined above is consistent with primordial element abundance measurements for a large region of oscillation parameter space. It turns out that this parameter space region requires some small interfamily mixing.

The challenge for observational cosmology, then, is to pin down cosmological parameters precisely enough to test early universe neutrino physics in some detail. Continuing primordial element abundance measurements will help, but much dramatic new information is likely from the cosmic microwave background anistropy measurements promised by the future MAP and PLANCK satellite missions.[35]

4. Conclusion

The Exact Parity Model predicts that if ordinary neutrinos mix with their mirror partners, then they mix maximally. This has been proposed as a very natural and simple explanation of the very large mixing angle deduced from atmospheric ν_μ disappearence measurements. In addition, maximal oscillations of the ν_e into its mirror partner are well motivated by most of the solar neutrino data. With small interfamily mixing switched on, the LSND anomaly can be explained by ordinary $\nu_e \to \nu_\mu$ oscillations. The EPM offers a theoretically elegant solution to all of the neutrino puzzles within a model that had as its original motivation the retention of the full Lorentz Group as an exact symmetry of Nature. The model also has some very interesting consequences for early universe cosmology, particularly the process of Big Bang Nucleosynthesis. In addition to the important results that continue to be produced by SuperKamiokande, we await with interest upcoming experiments – such as K2K, SNO, KAMLAND and others – that will provide further crucial tests of the Exact Parity Model.

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